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INVESTIGATION OF SERIES OF CONSECUTIVE PAIRS  
OF LARGE EARTHQUAKES IN DIFFERENT SEISMIC  
REGIONS

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**Abstract**

Series of relatively large earthquakes in different regions of the Earth were studied. The chosen regions are of high seismic activity and have good contemporary networks for recording of the seismic events along them. The main purpose of this investigation is the attempt to describe with mathematical expression the seismic process in space and time. Statistical distributions of the distances and the times between consecutive earthquakes were considered. Based on the conducted studies, a characteristic function was found, that describes the statistical distribution of the parameters of consecutive seismic events best. The main aim of such approach is to propose formal criteria for recognition of different active tectonic areas, as well as a quantitative evaluation of the reliability of this recognition.

**Key words:** earthquakes, epicentre distributions, pair analysis, interevent distances and times distribution

**1. Introduction.** The study of the spatial and time distribution of earthquakes has fundamental importance for understanding the physics of earthquake generating process, including the regional and local tectonic processes. The non-random spatial pictures of seismicity contribute to the determination of subsurface fault planes, the description of earthquake source parameters and fault kinematics in different seismic areas. The time distribution of earthquake occurrence is often a non-random process. Examples of such non-randomness are swarms of

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earthquakes, aftershock sequence after the main shock, identification of seismic “gaps” and/or migration of seismic activity in the region [1, 2]. Statistical studies of earthquake catalogue parameters and particularly the distance and time between subsequent events for different seismic regions could help in differentiation of the regions by introducing specific statistical criteria or could confirm eventual similarity of the seismic process in the studied regions if such differentiation is not adequate. A variety of characteristic functions was tested in order to approximate the empirical statistical distributions of distances and time intervals between subsequent seismic events. The analysis of the fitted distributions revealed that there is one function, giving the best fit. The method described below was applied to series of relatively strong earthquakes in several seismogenic regions.

**2. Methods.** In the present research, different catalogues published or available on the Internet, were used. The initial data were selected in such a way that they included data with a representative number of events and with a sufficient accuracy of the defined parameters of the earthquakes. The collected data before the year 1900 have too low precision and they were removed from the initially chosen catalogue. The data left (after 1900) cover a comparatively long period of time, during which there is enough collected information to describe the current seismicity of the examined zones.

Each catalogue was analysed to determine the minimal magnitude  $M_C$ , above which the catalogue could be considered complete. The freely available software programme Zmap [3] was utilized.

Statistical methods are very powerful in describing many processes. Application of such methods in seismology may be of great importance and might suggest new ideas about earthquake occurrence. The Poisson’s process describes relatively well many natural one-dimensional point processes, including earthquakes. It is a stationary process in which the number of events that occur in a certain time interval is independent of the number of the events in another time interval, when both intervals do not overlap [4]. The probability of occurrence of  $n$  events during a period of time  $\tau$  is determined by

$$(1) \quad P_n = \frac{(\lambda\tau)^n}{n!} \exp(-\lambda\tau),$$

where  $\lambda$  is the average number of events per unit time ( $\lambda = \text{const}$ ). The time intervals between the consecutive events are distributed exponentially, where the exponent is  $\lambda$  [5, 6].

The success in all attempts for studying the randomness of a series of main events depends on the ability to identify the aftershock events and their removal from the initial catalogue. “Methods of the moments” [7–9] from second or higher order developed for studying the spatial distribution of the earthquakes seem to be suitable for the aftershocks removal. Such methods were applied by REASENBERG [10]. The preliminary aftershocks removal in the current research was done by

means of the Reasenberg's spatial-time window realized in the Zmap software programme [3].

The seismic zones selected for this study are listed in Table 1. A specially developed programme (KOOR) by the author was used for the initial data treatment. According to the distances between the epicentres and the interevent times were calculated. After that the corresponding histograms of distances and interevent times distributions were constructed. Thus obtained empirical frequency distributions were modelled with different suitable functions. A software package TABLE CURVE specialized in making data regression and correlation analysis was used. The empirical frequency distribution of the distances was approximated by the following mathematical expression:

$$(2) \quad f(x) = c_x x^{p-1} (1-x)^{q-1},$$

where  $p, q$  are the parameters of the distribution and  $p > 0, q > 0$ , and the variable  $x, 0 \leq x \leq 1$ . The coefficient  $c_x$  is a normalizing factor such that  $\int_0^1 f(x) dx = 1$  [11]. Analogous procedure was applied to the histograms for the time intervals between the pairs of earthquakes. The analytical type of the used curve is

$$(3) \quad f(x) = c_t \theta \exp(-\theta t).$$

This is an exponential distribution with parameter  $\theta$  [4];  $c_t$  is also a normalizing factor.

**3. Results and discussion.** The chosen zones and the period of each catalogue are described in Table 1. The zones were chosen in "rectangular spatial windows". Their coordinates are indicated also there. The selection was done according to the zones recognition made by the seismologists from North America, Canada and Russia.

As a last step, from the catalogues with removed aftershocks series relatively strong events in each zone were chosen. The magnitude threshold was  $M_{th} = 4.0$ , and for some zones a second series with  $M_{th} = 5.0$  was also studied. Table 1 shows the number of events that were included in the present research.

The described method for approximation of the empirical frequency distributions of the distances and time intervals between the consecutive pairs of earthquakes was applied to the final data. The derived parameters from the approximation with the equations (2) and (3) together with their standard errors are also listed in Table 1.

As the results show, the distance distribution between the consecutive earthquakes in the studied zones have very similar behaviour (Fig. 1). In general, they are unimodal in most of the cases and only the position of the maximum and the slopes of both sides differ. The only zone, which differs significantly is the Cascadia zone (Fig. 1), where the approximating curves are decreasing with increasing

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Parameters of the approximating functions for the distributions of distances –  $c_x$ ,  $p$ ,  $q$  from function (2) and time intervals –  $c_t$ ,  $\theta$  from function (3), for series of large consecutive events.  $\theta_{av}$  is mean rate calculated for each zone,  $N$  is the number of events,  $M_{th}$  is magnitude threshold chosen for investigation zones

Zone	$N$	Period	$M_{th}$	$c_x$	$p$	$q$	$\theta_{av}$ num/day	$\theta$ num/day	$c_t$
1. Imperial Valley, California (32.5–33.3°N, 115.0–5.8°W)	53	1906– 1974	4.0	1.700 (±0.461)	1.749 (±0.135)	2.543 (±0.236)	0.0021	0.0021 (±0.0009)	343 (±92)
2. West Quebec, Canada (44.0–47.5°N, 72.0–77.5°W)	45	1903– 1992	4.0	1.167 (±0.967)	1.845 (±0.478)	2.151 (±0.613)	0.0014	0.0014 (±0.0001)	514 (±36)
3. Morgan Hill, California (37.1–37.4°N, 121.5–121.8°W)	29	1864– 1996	4.0	0.506 (±0.163)	1.340 (±0.143)	2.558 (±0.307)	0.0006	0.0005 (±0.0001)	1044 (±110)
4. Cascadia, Canada (47.8–52.0°N, 121–131°W )	122	1964– 1999	4.0	0.197 (±0.216)	0.677 (±0.430)	1.905 (±1.148)	0.0093	0.0088 (±0.0007)	93.2 (±0.5)
	29	1918– 1996	5.0	0.084 (±0.066)	0.341 (±0.347)	0.830 (±0.466)	0.0010	0.0012 (±0.0001)	1069 (±42)
5. Offshore region, Canada (48–52°N, 127.5–133.0 °W)	195	1959– 1991	4.0	1.727 (±0.623)	1.958 (±0.168)	3.616 (±0.390)	0.0164	0.0159 (±0.0005)	62 (±1)
	98	1917– 1991	5.0	1.461 (±1.099)	1.697 (±0.291)	4.309 (±1.030)	0.0036	0.0044 (±0.0006)	306 (±18)

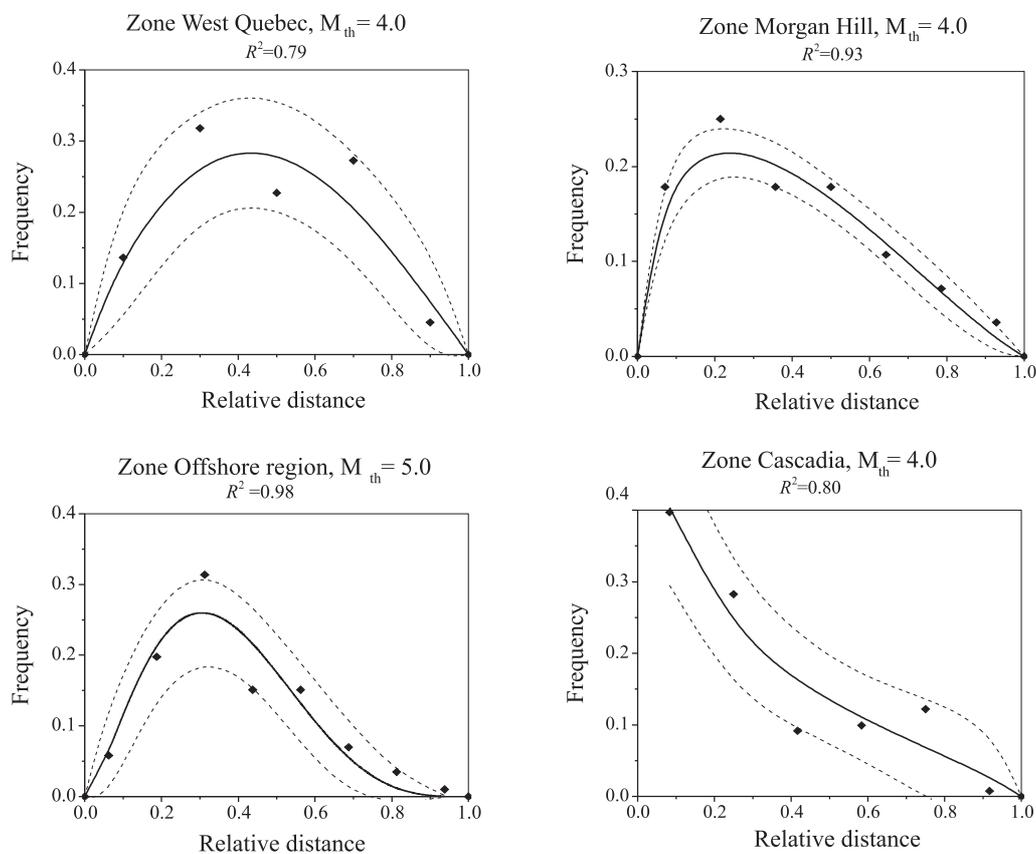


Fig. 1. Series of large earthquakes – approximations for the distributions of distances. At each figure the 90% confidence interval of the curve is presented in dash line. (The distances in these graphics are in relative measures – they are obtained as the real ones are divided by maximum distance, calculated for each zone.)

distances. A similar curve was obtained for other zone (Hellenic Arc), studied earlier by RAKOVA [12]. In our study, no difference in the approximating curves for the zones of the different series with a magnitude threshold  $M_{th} = 5.0$  was observed in comparison with those having  $M_{th} = 4.0$ . When series with magnitude threshold  $M_{th} \geq 5$  were investigated, the number of events decreased. When such series of the data for Cascadia and Offshore region were fitted with the equation (2), the points were scattered around the curve and the 90% confidential interval became relatively wide.

The coefficient of determination  $R^2$  quantifies the fit goodness of it has values ranging from 0.79 to 0.99 for the investigated zones (Fig. 1).

The distances in Fig. 1 are relative – they were obtained as a ratio between the current real value and the maximum distance, calculated for each zone. The results show that the maximal number of distances belong to the interval 0.2–0.4.

Offshore region was one of the largest zones. The distance distribution for it has a maximum at about 150 km. For the smaller West Quebec zone, Canada, that maximum is at about 220 km. For the Imperial Valley and Morgan Hill zones which were small in size the maximums of the curves were at about 40 and 15 km, respectively. This means that the distances in absolute units (km) may vary substantially.

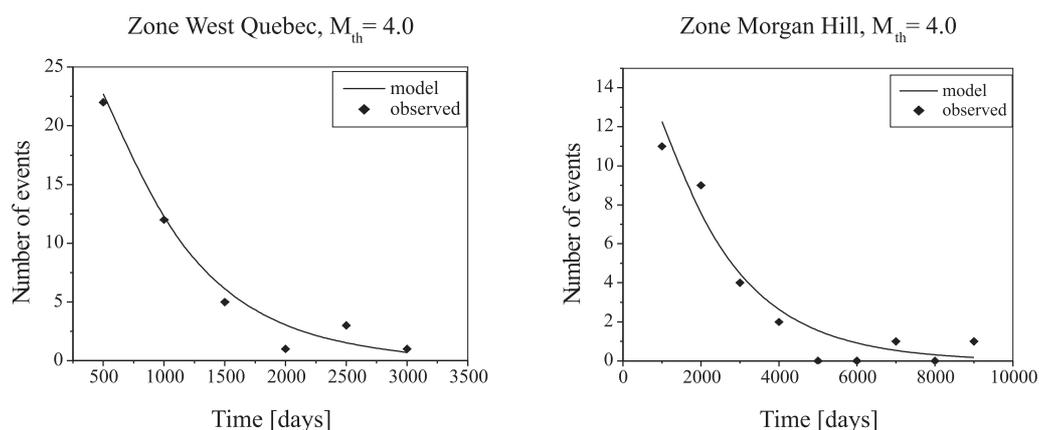


Fig. 2. Frequency of occurrence of time intervals between consecutive earthquakes: observed (presented with points) and model exponential (with smooth line) distribution with exponent  $\theta = \theta_{av}$  for some of the studied seismic zones

The approximating function (3) was applied to the empirical frequency interevent time distributions between consecutive pair of events. The obtained parameter values of the approximating function are listed in Table 1. In some of the cases, there are deviations between the values of the main rate of the events for a given zone ( $\theta_{av}$ ) and the derived from the approximation of the real frequency distribution – parameter  $\theta$  (by TABLE CURVE). In most of the cases,  $\theta_{av}$  values lie in the error limit of  $\theta$ . The deviations might originate from the fact that some of the samples had small number of events.

The deviations show that it is better to compare not the approximations but the real and the model distributions. The model distribution is exponential having the real main rate  $\theta_{av}$  (defined for each investigated zone) as an exponent. Such distribution of the time intervals between consecutive events is expected for the independent earthquakes, which follow the stationary Poisson's process. Figure 2 presents the comparison between the real and the model exponential times distributions of the subsequent events.

The obtained parameters of the approximating function (2) with their standard errors are shown in Fig. 3. They have relatively narrow error intervals, except

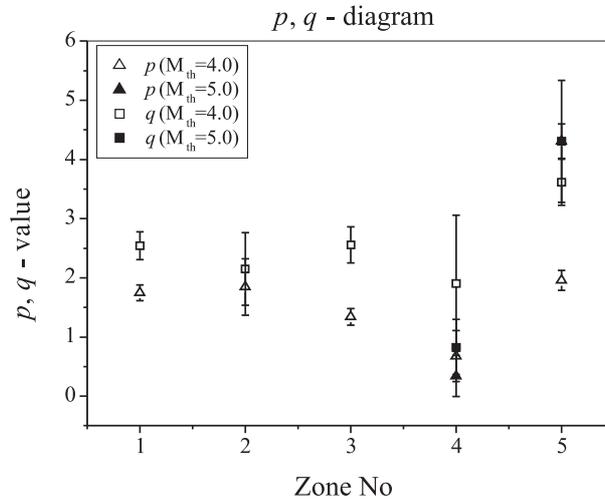


Fig. 3. Diagram of the parameters values  $p$  and  $q$  for the approximating function (2) for the studied zones. The standard error bars, received by the approximations are shown on the diagram. (On the diagrams the values for the series with  $M \geq 5.0$ , separated for some zones are shown, too.)

for zones Cascadia and Offshore region. Hence, the results for these zones show lower reliability. The agreement between the empirical and model distribution (2) has 5% significance level, according to  $\chi^2$  criterion.

The normalizing parameter  $c_x$  has lesser influence than  $p$  and  $q$ . Obviously, its precision is influenced by the degree of the points scattering.

**4. Conclusion.** The results can be summarized as follows: The proposed methodology for study of the distance distributions between consecutive large earthquakes from different seismogenic zones shows that there is approximating function suitable for such descriptions. The results suggest that the empirical distributions are close to unimodal curves. The position of the maximum of those curves differs, which could be attributed to different spatial locations of the events. Only few of the investigated cases showed different type of distribution (Fig. 1). The closeness of the empirical distributions with some mathematical functions allows to compare the investigations of the different zones. Moreover, they could be compared with other models and an estimation about their closeness could be derived.

The frequency distribution of time intervals between consecutive events is exponential, in good agreement with the theoretical exponential distribution with observed mean rate of earthquake occurrence (Fig. 2). The spatial and temporal clustering of aftershocks is a dominant non-random element of seismicity, so that when aftershocks are removed, the remaining activity can be modelled (to a first approximation) as a Poisson's process. This means that the earthquakes

in the investigated zones (and in the selected magnitude thresholds) are independent of each other and randomly distributed in time. This refers at least to the investigated periods for the selected seismic zones.

The methodology allows a quantitative evaluation of the accuracy of the functional parameters obtained.

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