# Spatial and Temporal Properties for Same Series of Relatively Strong Earthquakes

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**Abstract.** Series of relatively large earthquakes in different regions of the Earth are studied. The regions chooses are of a high seismic activity and has a good contemporary network for recording of the seismic events along them. The main purpose of this investigation is the attempt to describe analytically the seismic process in the space and time. We are considering the statistical distributions the distances and the times between consecutive earthquakes (so called pair analysis). Studies conducted on approximating the statistical distribution of the parameters of consecutive seismic events indicate the existence of characteristic functions that describe them best. Such a mathematical description allows the distributions of the examined parameters to be compared to other model distributions.

**Keywords:** Earthquakes, epicenter distributions, pair analysis, interevent distances and times distribution.
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## INTRODUCTION

The study of the spatial and time distribution of the earthquakes has fundamental importance for understanding of the source physics of earthquakes, including the regional and local tectonic processes. The non-random spatial pictures of the seismic activities allow subsurface fault planes to be determined and the measures of the earthquake destruction to be described. The time distribution of earthquakes occurrence frequently is nonrandom process. This can be seen when there are swarms of earthquakes, a sequence – main shock-aftershocks occurred or seismic "gaps" and/or migration of seismic activity can be observed [1, 2]. The study of the parameters of the statistic distributions, and more precisely distance and time between the consecutive events for different seismic regions can help to be found properties of these distributions. These studies allow verification of the hypothesis of independence, which they often derive from. The studies in approximation of statistical distributions of the parameters of consecutive seismic events that were carried out showed existence of characteristic functions that describe them best.

Series of relatively strong earthquakes from some seismic regions are taken into consideration.

## METHODS

The objective discrimination between non-randomness and randomness is a major challenge in the application of statistical methods to study of earthquake occurrence. Statistical models describing the time distributions range from of a simple Poisson process to more elaborate models including clusterization [3]. In contrast, the statistical study of the earthquake spatial distribution seems to lag behind - the two- or three-dimensional model needs to describe the spatial occurrence the earthquakes, which is done harder than in the case of a one-dimensional time model. Some authors base the need to check the earthquake spatial distribution on statistical ground. The method of statistical moments has been applied for this purpose in a series of studies [1, 2, and 4]. From a probability theory point-of-view the seismic regime is best described either as a sequence of random events in a multi-dimensional phase space, or as a random point process, as considered in [2].

The application of the statistical methods in the research of time distribution of earthquakes is very important in seismology because such researches can suggest new ideas for the patterns of earthquake appearance. The Poisson's process describes relatively well many natural one-dimensional point processes, including earthquakes. This is a

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stationary process, where the numbers of events that have occurred in a certain interval of time do not depend on their number in other interval, which does not coincide with the first interval [5]. The probability of occurrence $n$ number of events for period $\tau$ is determined by

$$P_n = \frac{(\lambda\tau)^n}{n!}\exp(-\lambda\tau)\,, \tag{1}$$

Where $\lambda$ = const is the average number of events for a time unit or an indicator for their appearance, too. The time intervals between the consecutive events are exponentially distributed by indicator $\lambda$ [6, 7]. From point of view of evaluation of the seismic hazard the sequence of main large events has the great significance. In that sense the main question in their study is their distribution in time. The "simple" Poisson's process, as a stochastic process is used for description of the independence of such events. If the Poisson's distribution of the earthquakes is confirmed, then the occurrence will be included as an internal feature of seismic regime.

The selected for these study seismic zones are listed in Table 1. The coordinates were taken from different catalogs published and available on the Internet. At least part of the catalogs contain earlier events (usually large earthquakes), the catalogs chosen for examination cover the time after 1900. The selection is made in order the catalogs to be more complete and homogeneous. A research is made on all data for determination of minimal magnitude $M_C$, towards which the catalog could be considered complete (with the program Zmap [8]). This provides a sufficient accuracy in defining the epicenter coordinates and besides it is a relatively long period during which information for description of the current seismic activity in the regarded zones is accumulated.

The success of all attempts for studying the randomness of a series of main events depends on the ability the aftershocks to be identified and their removal from the catalogue. The development of the studies of the spatial distribution of the earthquakes under the so-called "methods of the moments" [9, 10] from second or higher order seem to be suitable for removal of the aftershock series, too, as *Reasenberg* [4] does for example. The preliminary filtration of aftershock events is applied where the Reasenberg's spatial-time window, realized in the program Zmap [8] is used.

The specially developed by then program the distances between the epicenters and then the interevent times for each pair consecutive earthquakes from the initial seismic catalogue are calculated. The corresponding histograms of distribution for the distances and interevent times are made by the data, received from the initial processing. The biggest attention was paid on the approximation of thus obtained empirical frequency distributions with suitable function. A program package Table Curve specialized in making regression and correlation analysis of data is used for that aim. An approximation of the distributions of the relative frequency of occurrence for distances between the consecutive shocks is the mathematical expression given by

$$f(x) = c_x x^{p-1}(1-x)^{q-1}\ . \tag{2}$$

The parameters of the distribution $p, q$ are $p > 0, q > 0$, and the variable $x$: $0 \le x \le 1$. The coefficient $c_x$ plays the role of a "normalizing factor", or such as

$$\int_0^\infty f(x)\,dx = 1 \qquad [11].$$

Analogous procedure is used at the approximation of histograms for the time intervals between the pairs of earthquakes. The analytical type of the used curve is:

$$f(t) = c_t\theta\exp(-\theta t)\,, \tag{3}$$

This is an exponential distribution with parameter $\theta$ [5]. The role of $c_t$ is also that of a "normalizing factor".

Model distribution of distances between consecutive earthquakes, for which is accepted the hypothesis of independence, can be obtained empirically in the following way. We consider the statistical distributions of the distances between each two consecutive earthquakes. If any regularity is to be searched, they must be compared to model sequences of pure random character. To retain the highly non-uniform distribution of earthquakes in space, model sequences or parameter distributions might be developed by randomizing of the catalog, i.e. random reordering of events in time and space. Thus, non-random features (if any) will be destroyed and only events space distribution will influence the randomized catalog. Then after each disorder we should obtain random samples of all possible values of investigated parameters. But the whole information for events space distribution could be taken into consideration if all possible parameter values are used and not a simple of them. And that is the model distribution was developed in this investigation instead of using a randomizing procedure.

The distances between epicenters were calculated for each pair of consecutive events from the source earthquake catalog. The corresponding spatial distribution histograms were constructed using initially processed data there from. The earthquake pairs were formed by taking the distance $r$ between consecutive earthquakes only. Thus if $N$ is the number of the events, ($N$-1) pairs are formed. Using catalog data, the model distribution of distances $r$ between epicenters of all pairs of successive events is used for characterization the process of earthquake spatio-temporal realization:

$$P_i = N_i / N^{all}, \tag{4}$$

Where $N_i$ is the number in $i$-th histogram interval and $N^{all} = N(N-1)/2$ is number of all possible values. Then each expected values $N_i^{E} = (N-1).P_i$ is compared to observed own $N_i^{0}$ in equal histogram intervals [9].

Model distribution of distances between consecutive shocks can be obtained using a generator of random numbers, which can be viewed as coordinates of “earthquakes” in rectangular area. The usage of that generator corresponds to the hypothesis of uniform surface distribution of points: the probability of an occurrence of an event in every point is equal. The model distribution is obtained after calculating the distances between consecutive events. The possibility of obtaining a model distribution is used in the present study. A FORTRAN programme is prepared for the purpose, with which an equal number of points is generated on a rectangular areas. The sizes of the areas coincide with the sizes of the studied seismic zones. The purpose of this is to compare these models of uniform distribution of independent events to the real distributions of distances between consecutive earthquakes catalogs.

## RESULTS AND DISCUSSION

In Table 1 the chosen zones and the period of each catalogue are described. For the sake of convenience the zones are chosen in “rectangular spatial windows”. Their coordinates are indicated in the same table. The selection is done according the zones’ recognition made by the seismologists in North America, Canada [12-16].

On the catalogues of the studied seismic zones additional filtration is made, in order series of relatively strong events in each zone to be separated. Lower magnitude threshold $M_{th} = 4.0$ is chosen, and some zones have also a second series with $M_{th} = 5.0$. Table 1 shows also the number of the events that are included in the research.

The described methodology for approximation of the empirical frequency distributions of the distances and time intervals between the consecutive pairs of earthquakes is applied for thus received data. Table 1 gives also the obtained parameters of the approximating curves (2) and (3) together with their standard errors. It is typical for the distribution of the distances between the consecutive earthquakes in the studied zones the approximating curves to have one and the same character. In general they are unimodal in most of the cases and only the positions of the maximums are different. The approximating curves for zone Cascadia are different – they are decreasing when distances increase. A similar curve is received also for other zone, studied earlier – Hellenic Arc, described in [17].

The separation of series of large earthquakes leads to decrease in the number of events. When there is fitting with equation (2), in some cases – zones West Quebec, Cascadia, Toktogul – the points are scattered around the curve.

The comparison between the described models of series of independent earthquakes and the suggested model function, which approximates the real distributions of distances, shows the following:

- For the series of zones Garm ($M \geq 4.0$) and Toktogul there is a coincide of the different models, which shows that the events are independent and correspond to the uniform distribution of the epicenters in the zone
- For the series of zones Imperial Valley, West Quebec, Garm ($M \geq 5.0$) and Offshore region there is a good correspondence between the empirical model and the fitting the real distributions curve, which confirms the assumption of independence. The distributions of the epicenters in these zones do not correspond to the uniform distribution.
- For zone Cascadia the empirical and the real distributions concur, which confirms the assumption of independence; the distribution of epicenters in this zone differs from uniform distribution.

**TABLE 1.** Parameters of the approximating functions for the distributions of distances– $c_x$, $p_x$, $q_x$ from function (2) and time intervals – $c_t$, $\theta$ from function (3), for series of strong consecutive events. $\theta_{av}$ is mean rate calculated for each zone, N is the number of event, $M_{th}$ is magnitude threshold chosen for investigation zones.

| Zone | N | Period | $M_{th}$ | $c_x$ | $p_x$ | $q_x$ | $\theta_{av}$ num/day | $\theta$ num/day | $c_t$ |
|---|---|---|---|---|---|---|---|---|---|
| **1**.Imperial Valley, California<br>(32.5-33.3°N,115.0-115.8°W) | 53 | 1906-1974 | 4.0 | 1.700 (±0.461) | 1.749 (±0.135) | 2.543 (±0.236) | 0.0021 | 0.0021 (±0.0009) | 343 (±92) |
| **2**.West Quebec, Canada<br>(44.0-47.5°N, 72-77°W) | 45 | 1903-1992 | 4.0 | 1.167 (±0.967) | 1.845 (±0.478) | 2.151 (±0.613) | 0.0014 | 0.0014 (±0.0001) | 514 (±36) |
| **3**. Garm, Asia<br>(38.5-39.5°N, 70.0-71.6 °E) | 122 | 1962-1999 | 4.0 | 1.244 (±0.846) | 2.012 (±0.386) | 2.503 (±0.536) | 0.0089 | 0.0108 (±0.0008) | 100 (±4) |
|  | 43 | 1924-1997 | 5.0 | 0.378 (±0.268) | 1.056 (±0.295) | 2.116 (±0.703) | 0.0016 | 0.0018 (±0.0002) | 481 (±34) |
| **4**. Cascadia, Canada<br>(47.8-52.0°N, 122-131°W ) | 122 | 1964-1999 | 4.0 | 0.197 (±0.216) | 0.677 (±0.430) | 1.905 (±1.148) | 0.0093 | 0.0088 (±0.0007) | 93.2 (±0.5) |
|  | 29 | 1918-1996 | 5.0 | 0.084 (±0.066) | 0.341 (±0.347) | 0.830 (±0.466) | 0.0010 | 0.0012 (±0.0001) | 1069 (±42) |
| **5.** Offshore region, Canada<br>(48-52°N, 127.5-133.0 °W) | 195 | 1959-1991 | 4.0 | 1.727 (±0.623) | 1.958 (±0.168) | 3.616 (±0.390) | 0.0164 | 0.0159 (±0.0005) | 62 (±1) |
|  | 98 | 1917-1991 | 5.0 | 1.461 (±1.099) | 1.697 (±0.291) | 4.309 (±1.030) | 0.0036 | 0.0044 (±0.0006) | 306 (±18) |
| **6**. Toktogul, Asia<br>(39.2-43.5°N, 69.2-76.0°E) | 196 | 1930-1960 | 4.2 | 0.846 (±0.605) | 1.785 (±0.359) | 2.646 (±0.642) | 0.0176 | 0.0200 (±0.0009) | 61.6 (±1.4) |

The coefficient of determination $R^2$ quantifies goodness of fit and for investigated zones is between 0.79 and 0.99.

The observed frequency distribution of time intervals between consecutive earthquakes is shown in Figure 2. The approximations of these real distributions with exponential function show good agreement: the coefficient of determination $R^2$ changes between 0.86 and 0.98. Because the periods that covered the regarded data from the catalogues are different, the obtained from the approximations values $\theta$ vary. Distribution of observed earthquake interevent time for the declustered earthquake catalogs plotted along the theoretical exponential distribution for catalogs with the observed mean rate of earthquake occurrence. The mean rate $\theta_{av}$ is calculated for each zone (tab.1). In most of the cases there is not a significant difference between the values of the parameter $\theta$, received by the approximating procedure and the calculated mean rate $\theta_{av}$ – the differences are reasonable within the mean standard errors. The main advantage of this fitting procedure is the property to calculate the errors of the fitting curves. This means that is possible to assess the dispersion of the points around the curve, which physically means the precision of the approximation. That is why the line approximating the real distribution of the times between the consecutive events almost coincides with that of the exponential distribution with indicator equal to $\theta_{av}$ for each zone (Figure 2). At Figure 2 the results of the approximation only for some zones are shown, because the curves are similar.

The change of the parameters of the approximations of the distributions according to the distance in these cases is within the limits:

- For $p_x$ – between 0.3 and 2.1; for $q_x$ – between 0.8 and 4.4.

As it can be seen from the Table 1, the obtained parameters have relatively narrow intervals of the errors. The most significant errors are these for the zones Cascadia and Offshore region, Canada, and in a lesser extent those for Garm, Asia. This means that the level of reliability of the results from the approximation of the distribution of consecutive distances in this zone is lower. The agreement between the empirical and model distribution (3) has different level according to the $\chi^2$ criterion. In most of the cases the level of significance is 5 %.

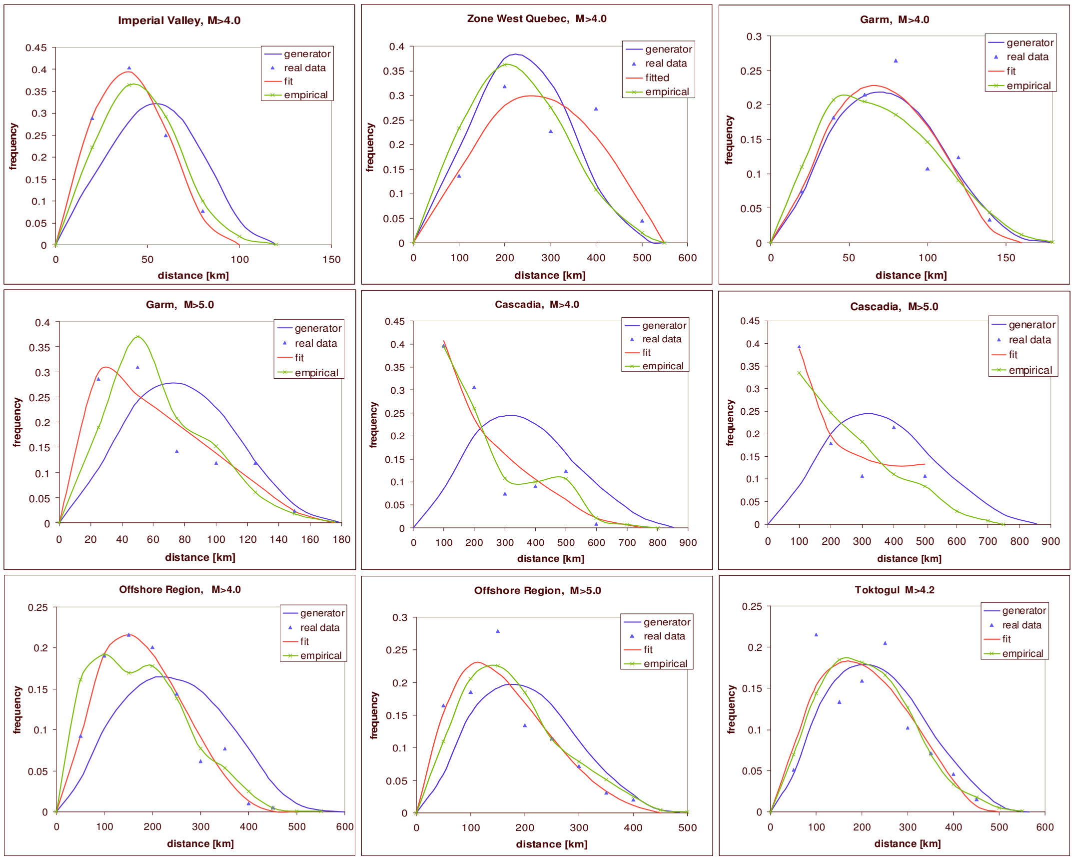

**FIGURE 1.** Series of strong earthquakes – approximations for the distributions of distances between consecutive earthquakes and comparison with an empiric model and a uniform distribution model.

The parameter $\theta$ has a high accuracy and range over a wider diapason of change, which reflects the difference of the periods that the catalogues cover. The values only for zones Cascadia and Offshore region, Canada are an exception. The strongest declination between the real distributions of the times between the shocks and the approximating exponential functions is observed for these zones. The normalizing parameters $c_t$ and $c_x$ have a lesser relative influence. Obviously their precision is influenced by the degree of diffusion of the points from the graphics.

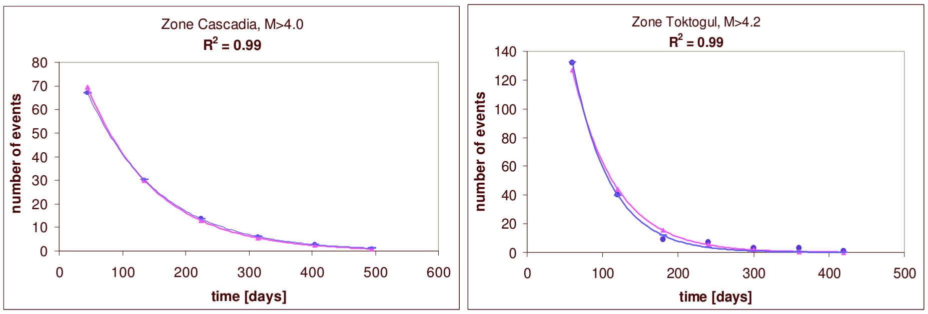

**FIGURE 2.** Frequency of occurrence of time intervals between consecutive earthquakes: observed (presented with points) and model exponential (with smooth line) distribution with indicator $\theta = \theta_{\text{av}}$ for the zone, as well as the line fitting the observed data (with dash line), for some of the studied seismic zones.

## CONCLUSION

The physical meaning of the results obtained could be summarized as follows:

- The distances intervals show curves with unimodal shape. This could be interpreted as the existence of the predominant distances between the consecutive seismic events with certain magnitude – Fig.1. The empirical distributions of these distances can be described by suitable mathematical expression.
- The frequency of occurrence of time intervals between consecutive events is exponentially related to these intervals, in good agreement with the theoretical exponential distribution with the observed mean rate of earthquake occurrence – Fig.2. The spatial and temporal clustering of aftershocks is dominant nonrandom element of seismicity, so that when aftershocks are removed, the remaining activity can be modeled (as first approximation) as Poisson process. This means that the earthquakes in the investigated zones (and in the selected magnitudes thresholds) are independent of each other and randomly distributed in time. This is hold at least for the investigated periods for selected seismic zones.

The proposed methodology for study of the distributions of distances and time intervals between consecutive large earthquakes from different seismogenic zones shows that there are approximating functions suitable for such descriptions. The obtained results confirm that the used approximating functions show differences in the spatial distribution of the events from the different seismogenic zones. The methodology allows a quantitative evaluation of the quality of the obtained functional parameters to be given.

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