MODEL STUDIES OF THE ENERGY DISTRIBUTION OF SEISMIC SEQUENCES IN THE UPPER-THRACIAN DEPRESSION

***Dragomir Gospodinov*[1], *Elisaveta Marekova*[2]**
[1]Geophysical Institute, BAS; drago_pld@yahoo.com
[2]Plovdiv University; eligeo@pu.acad.bg

Abstract. *Magnitude behaviour of earthquakes in the Upper-Thracian depression has been stadied by anallyzing the differences between magnitudes of subsequent events. A theoretical distribution of differences has been developed on on the assumption of independence. This assumption implies the magnitude of a shock not to be dependent on the history of the process. The model distribution has been compared to the real distribution of magnitude differences between subsequent events for the Upper-Thracian depression catalog.*

Introduction

Earthquake interaction in a seismic region is the purpose of many researchers' studies Different aspects of the seismic process are being examined by the investigators as the temporal behaviour of earthquakes, their possible spatial correlation and the distribution of events by magnitude. Also models are being developed to describe the effects of interaction between earthquakes – between foreshocks and main events and the triggering of aftershocks by strong earthquakes. Stochastic models like the modified Omori formula (MOF) (Utsu, 1961), the epidemic type aftershock sequence (ETAS) model (Ogata, 1988) and the restricted epidemic type aftershock sequence (RETAS) model (Gospodinov & Rotondi, 2004) are widely applied to depict the temporal distribution of triggered events.

Another approach to examine the dynamics of the seismic process as a sequence of events, except using stochastic models, is to analyze parameters characterizing the subsequent occurrence of earthquakes in a sequence. Such parameters can be the spatial distance (Gospodinov & Christoskov, 1988), (Kagan & Knopoff, 1980) and the time intervals between subsequent shocks.

In our paper we study the energy distribution of earthquakes in a seismic region in Southern Bulgaria. Usually the distribution of earthquakes by their strength is analyzed by examining the recurrence formula of Gutenberg-Richter (1956) presented by

$$\lg N = a - bM \tag{1}$$

where N is the number of earthquakes with magnitudes bigger or equal to M and b is a parameter determining the slope of this linear relation. This formula, however, considers events' magnitudes regardless of their order of occurrence and in that way we loose information about possible relations between magnitudes of subsequent shocks. For this reason we decided to study the magnitude difference $\Delta M = M_{i+1} - M_i$ between two subsequent earthquakes. We consider this parameter a random variable constructed as a difference of two random variables M_{i+1} and M_i and we compare its distribution to the real magnitude differences.

ΔM statistical model

We suggest an initial hypothesis that the magnitude of an earthquake is not dependent on the magnitude of the previous event, so it can be considered as a random variable. Its distribution can be obtained following formula (1).

Assuming this hypothesis to be true we may elaborate the distribution of $\Delta M = M_{i+1} - M_i$. Let us have a sequence of $N+1$ events. From this sequence we obtain N differences ΔM between subsequent magnitudes. If we denote

$$Z = M_{i+1} - M_i = Y - X \qquad i = 2,\ldots,N \tag{2}$$

then we reduce the problem to finding the distribution of the difference of two independent, identically distributed random variables (Blom, 1989) with density distribution functions obtained after transforming eq.1 so that the density follows an exponential distribution

$$\begin{aligned} g(x) &= \beta\exp(-\beta x)/C \qquad x \in A = [M_L, M_H] \\ g(y) &= \beta\exp(-\beta y)/C \qquad x \in A = [M_L, M_H] \end{aligned} \tag{3}$$

Here M_L and M_H denote the lower and the upper magnitude cut-off of the events in the catalogue and

$$C = [\exp(-\beta M_L) - \exp(-\beta M_H)] \tag{4}$$

So, knowing the distributions of the two magnitudes (as random variables) and assuming that they are independent, we may obtain the distribution function of their difference

$$F(z) = \Pr(Z < z) = \iint_{G(Z<z)} f(x,y)\,dxdy = \iint_{G(Z<z)} g(x)g(y)\,dxdy \tag{5}$$

where

$$Z = Y - X \qquad Z \in [M_L - M_H, M_H - M_L] \tag{6}$$

Substituting (3) in formula (4) and solving the integral, we obtain the distribution function of the magnitude difference for two cases (Gospodinov & Rotondi, 2001)

(i) $Z \in [M_L - M_H, 0]$

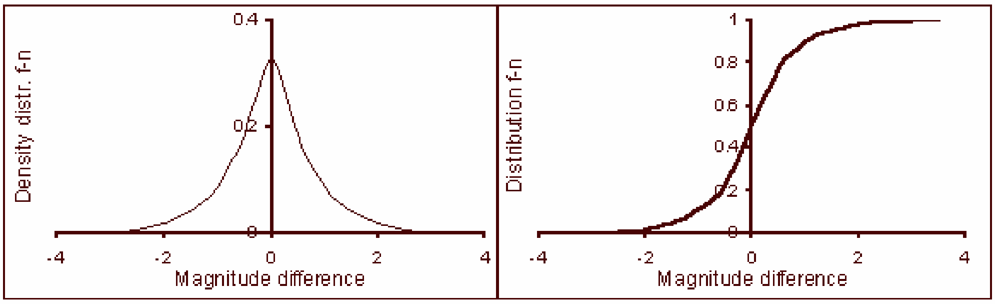
$$\begin{aligned} F(z) &= \int_{M_L - z}^{M_H} [\int_{M_L}^{z+x} \frac{\beta\exp(-\beta y)}{C} dy] \frac{\beta\exp(-\beta x)}{C} dx = \\ &= \{0.5\exp[-\beta(2M_L - z)] - \exp[-\beta(M_H + M_L)] + 0.5\exp[-\beta(2M_H + z)]\ \}/C^2 \end{aligned} \tag{7a}$$

(ii) $Z \in [0, M_H - M_L]$

$$\begin{aligned} F(z) &= 1 - \Pr(Z > z) = 1 - \int_{M_L}^{M_H - z} [\int_{z+x}^{M_H} \frac{\beta\exp(-\beta y)}{C} dy] \frac{\beta\exp(-\beta x)}{C} dx = \\ &= 1 - \{0.5\exp[-\beta(2M_L + z)] - \exp[-\beta(M_H + M_L)] + 0.5\exp[-\beta(2M_H - z)]\ \}/C^2 \end{aligned} \tag{7b}$$

So, formulae (7a,b) provide the distribution function of $\Delta M = M_{i+1} - M_i$ in the case that our hypothesis for independent magnitudes of subsequent magnitudes is true. As can be seen the distribution is a function of the lower and upper magnitude limits for the catalog and also of β where $\beta = b \ln 10$.

Transforming these formulae we can also get the density distribution function. Both of them are graphically presented on Fig.1. The density distribution function is plotted on Fig.7a and Fig.7b shows the form of the distribution function. One may see that the density distribution is symmetrical towards the zero, having a peak at this value. We use this model distribution to calculate the expected values after the basic hypothesis. Any statistically significant divergence of the real number of magnitude differences from the expected ones rejects our hypothesis according to its place and value.



a) b)

Fig.1 Distribution graphs of the magnitude differences $\Delta M = M_{i+1} - M_i$ of subsequent earthquakes in a catalogue if the magnitude of an event is considered to be independent of the previous shock. Then M_{i+1} and M_i are believed to be random variable with identical distribution following eq.(1); a)graph of the density distribution function of $\Delta M = M_{i+1} - M_i$; b) graph of the distribution function of $\Delta M = M_{i+1} - M_i$.

Data

The catalogue data for this study is compiled after a project of the Physical department of Plovdiv University 01-Ф-10 from 2001-2002. The region under exam is defined by vertices

41.7°N, 24.0°E 41.7°N, 27.0°E;
42.8°N, 24.0°E 42.8°N, 27.0°E

Several strong events have occurred in it the most damaging ones being in April 1928 with magnitudes M=6.8 and M=7.0. We have compiled the catalogue using different sources. As earthquake magnitudes in these sources were calculated after different scales we unified all values to M_S applying appropriate transitional formulae. The catalogue contains 3173 events. We have chosen a lower magnitude of M_L=3.5 above which the catalogue seems to be complete and in that way the number of earthquakes we analyzed is N=343.

Data analysis

When examining the magnitude distribution of earthquakes, the basic empirical law in seismology is the recurrence formula presented by eq.(1). It, however, does not give information about the sequential order of the magnitudes. There is another empirical law which depicts the relation between the magnitude of the main shock and the ones of the strong aftershocks following. It states that the average magnitude difference between the magnitude of the main earthquake and the magnitude of the strongest aftershock following it, is 1.2 (Bath, 1973)

$$M_{main\ shock} - M_{strongest\ \ aftershock} = 1.2 \qquad (8)$$

This so called Bath's law gives some insight about the magnitude relation main shock-strongest aftershock, but still, we have not found a proper physical hypothesis underlying an eventual model for the temporal distribution of the events' magnitudes in a sequence. Attempts have been made to identify some trend in successive magnitudes (decreasing or increasing) but we could not find any reliable results related to this aspect of the seismic process. In this situation the hypothesis that magnitudes of subsequent shocks are independent seems natural as an initial suggestion. So, the statistical model depicted by formulae (7a,7b) comes to represent this independence hypothesis and it is used to fit the real magnitude differences between subsequent earthquakes in the catalogue. The model was first applied to fit the data for the whole sample of N=343 earthquakes.

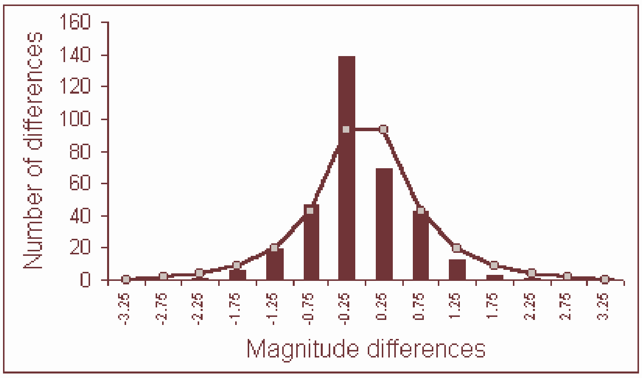


Fig.2 Histogram graphs of the magnitude differences $\Delta M = M_{i+1} - M_i$ of subsequent earthquakes in the catalogue for the whole period 1900-2001; continuous line – model values when the magnitude of an event is considered to be independent of the previous shock; columns – histogram values for the real magnitude difference between subsequent events in the catalog.

On Fig.2 we have plotted the histogram graphs of the magnitude differences $\Delta M = M_{i+1} - M_i$ of subsequent earthquakes in the catalogue for the whole period 1900-2001. The continuous line represents the model values when the magnitude of an event is considered to be independent of the previous shock and the columns give the histogram values for the real magnitude difference between subsequent events in the catalog. The distributions are plotted for difference ranges of 0.25.

As can be seen there is a certain discrepancy between model and real values for the histogram range -0.25÷0. This disagreement in negative values of the magnitude differences speaks about the existence of more gradual decrease of earthquakes' magnitudes in the sequence.

To verify whether this is not a random variation in the real data behaviour, we decided to divide the data into two samples – one containing the events up to the occurrence of the strong earthquakes in 1928 and another sample of events after these strong shocks.

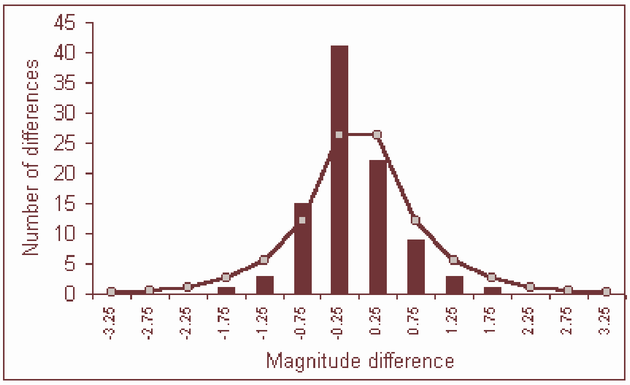


Fig.3 Histogram graphs of the magnitude differences $\Delta M = M_{i+1} - M_i$ of subsequent earthquakes in the catalogue for the period 1900-1928; continuous line – model values when the magnitude of an event is considered to be independent of the previous shock; columns – histogram values for the real magnitude difference between subsequent events in the catalog for this period.

The first sample contains N=96 earthquakes with $M_S \geq 3.5$. It is again fitted by the statistical model as calculated by eqs.(7a,7b) and the results are plotted on Fig.3. A similar disagreement between model and real data is identified for the same histogram range -0.25÷0, which comes as a proof that this divergence can hardly be considered as random.

The second sample after 1928 contains N=247 earthquakes and the results from the fitting of these data by the model are plotted on Fig.4. We observe the same kind of exceedence of the real data value over the model one for the range -0.25÷0, which supports the eventual non-randomness of this discrepancy.

To estimate the significance of this real data anomaly, we decided to randomly simulate 1000 magnitude sequences following the same recurrence law as the real data. We can then use the variations of these simulated data values as error bounds and to see whether the observed discrepancy is significant enough to stay out of these bounds.

We made the simulation following Ogata (1999)

$$F(M) = U_{n+1} \tag{9}$$

where $F(M)$ is the magnitude distribution function and U_{n+1} is a random number with uniform distribution. The distribution function can be presented using formula (1) or formula (3) and for our case we have chosen the former

$$F(M)=\frac{10^{-bM}-10^{-bM_L}}{10^{-bM_H}-10^{-bM_L}} \tag{10}$$

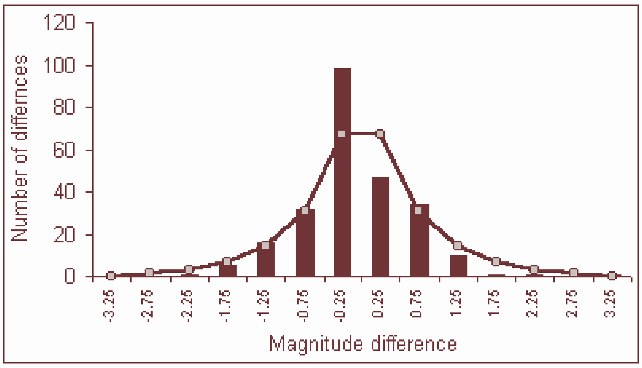


Fig.4 Histogram graphs of the magnitude differences $\Delta M = M_{i+1} - M_i$ of subsequent earthquakes in the catalogue for the period 1928-2001; continuous line – model values when the magnitude of an event is considered to be independent of the previous shock; columns – histogram values for the real magnitude difference between subsequent events in the catalog for this period.

Substituting eq.(10) in eq.(9) and solving the equation as regard to M, we can generate randomly a sequence of magnitudes which follow a recurrence law with the same b as the real data. We then construct again the histogram as on Fig.2,3,4 and select the minimum and maximum values obtained from the simulated data for each histogram range. These values were picked up after 1000 simulated magnitude sequences and they were used as error bounds. The results from this approach are plotted on Fig.5, where the error bounds are represented by dashed lines, the continuous line with blank rectangles stands for the model values after formulae (7a,b) and the columns with grey circles show the real data values. One can easily see that the anomaly stands well out (above) of the error bounds, which supports its statistical significance. The interpretation of this discrepancy between model and real data does not seem easy. We think that it may be considered as an indirect consequence of earthquake clustering. The existence of events clusters in which magnitudes decrease more gradually after a strong shock could lead to such a divergence from our hypothesis about magnitude independence of subsequent events.

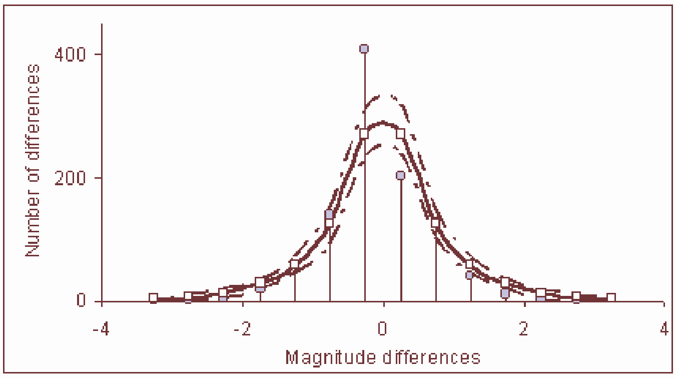


Fig.5 Histogram graphs of the magnitude differences $\Delta M = M_{i+1} - M_i$ of subsequent earthquakes in the catalogue for the whole period 1900-2001; continuous line – model values when the magnitude of an event is considered to be independent of the previous shock; dashed lines – error bound, determined by the maximum and minimum values for 1000 random simulations of magnitude sequences following certain recurrence law; columns – histogram values for the real magnitude difference between subsequent events in the catalog for this period.

Conclusions

In this paper we study the energy distribution of earthquakes in a seismic region in Southern Bulgaria. Usually the distribution of earthquakes by their strength is analyzed by examining the recurrence formula but it, however, considers events' magnitudes regardless of their order of occurrence. Doing so, one does not use the information about possible relations between magnitudes of subsequent shocks. That is why we chose a different approach - to study the magnitude difference $\Delta M = M_{i+1} - M_i$ between two subsequent earthquakes. We consider this parameter a random variable constructed as a difference of two random variables M_{i+1} and M_i and we develop a model statistical distribution of ΔM considering these two random variables to be independent. In fact this is our initial hypothesis, which we verify by fitting the model to the real data.

We first analyzed the whole data sample covering the period 1900-2001 and the fitting of the data to the model revealed a discrepancy in the ΔM -distribution for the histogram range -0.25÷0. This anomaly was confirmed for two smaller samples of the data; for the period before the strong earthquakes in 1900-1928 and another sample for the remaining time 1928-2001. We verified the statistical significance of this anomaly by simulating 1000 random magnitude sequences and using their maximum and minimum histogram values as error bounds. The identified anomaly surely stands out of these error bounds – the real data value is significantly bigger than the model one. We interpret this anomaly as an indirect result supporting the existence of earthquake clusters in which events magnitudes decrease more gradually after a strong shock. The ΔM -model seems applicable for analyzing also the aftershocks in a separate aftershock sequence which will be the purpose of our future investigations.

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