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FRACTAL PROPERTIES OF SIMULATED SEQUENCES IN 1D SPACE. APPLICATION TO EARTHQUAKE SEQUENCE ANALYSIS

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ABSTRACT

A fractal distribution requires that the number of objects larger than a specific size r has a power-law dependence on the size $N(r)=C/r^D \propto r^{-D}$, where D is the fractal dimension. The fractal dimension can be an integer and then it is equivalent to a Euclidean dimension. In general the fractal dimension is not an integer but a fractional dimension and there comes the origin of the term 'fractal'. The use of a power-law to statistically describe a set of events or phenomena reveals the lack of a characteristic length scale, that is fractal objects are scale invariant. A lot of studies point that earthquakes obey fractal statistics, so the fractal dimension can characterize them. In our study we have analyzed simulated sequences on a straight line. Then we have applied the same algorithms to study the fractal properties of earthquakes in space. The results have been compared to the ones of the simulated sequences.

Keywords: earthquakes, fractals, fractal dimension, correlation integral.

1. Introduction

Recently the use of fractal distribution [1] methods in geophysics has become very widespread.[2-4] Therefore, fractal analysis is one of the methods used for the study of self-similar phenomena and processes. Moreover, self-similarity in geophysics has a significant role [5,6], although it is not completely analyzed. Self-similarity means the existence of an invariant object structure in different dimensions of exploration. If the object is respective only of the spatial coordinates of the geophysical fields(for example, the relief or other peculiarities of the earth crust), not of time, then we can conclude that here we emphasize on spatial dimension, characterized by a certain size Δ. One of the characteristic features of the self-similar objects is the hierachic dependence of any of their elements of the dimension Δ. In order to define the dimension d of the distribution A in the n –dimensional space we take n –dimensional spheres with diameter Δ and the distribution A is covered with these spheres, so that the number of the covering spheres $N(\Delta)$ is minimal. Diminishing Δ, these spheres will write in space the structure of the distribution A more thoroughly. When $\Delta\to 0$ the number $N(\Delta)$ will tend to infinity. If $N(\Delta)$ is rising asymptomatically according to the hierarchic law, i.e.

$$N(\Delta) = c\Delta^{-d} + o(\Delta^{-d}),\ \Delta\to 0, \tag{1}$$

then the distribution на A is regarded to have d-dimension. Taking (1) into account we can infer the asymptomatic definition of the d -dimension:

$$d = \lim_{\Delta\to 0} \frac{\ln N}{\ln 1/N}. \tag{2}$$

The geometric sense of d shows that it is equal to the slope of the graphics $N(\Delta)$ in a double logarhythymic graph. The dimension can be an integer and then it is equivalent to the Euclidean dimension D_E (it is zero for a point, one for a straight line, two-for a square and three for a cube). As for corrugated lines and surfaces the d-dimension can be a fraction (always $d > D_E$) and there comes the origin of the term"fractal". Fractal objects are very widespread in nature.[1] One of the examples of the self-similar structure is the seismic regime, i.e. the number of earthquakes, explored as a point in time and space and connected with an additional parameter-energy. A classical example, proving the self-similarity of the seismic regime, is the Gutenberg-Richter relation, a fundamental law in seismology. The average number of earthquakes $N(M)$ with magnitudes greater than M, concerning an interval of time is defined in the following way:

$$\lg N(M) = a - bM, \qquad (3)$$

where a and b are parameters of the relation.

There are many articles on the spatial distribution of seismicity. The exploitation of the methods of fractal distribution as a number of earthquakes and microfractures in experiments with specimen [2,4,7] showed that number of origins have fractal dimensions smaller than the spatial dimensions. However, there came two difficulties that could not be easily overcome with the help of fractal distribution methods. The first one is the heterogeneous spatial structure of the seismic fields. Generally speaking, fractal distribution can be a local parameter and characterize the local structure of the self-similarity of a partiular place. The second one is the importance of the frequency of "visiting"this dimensions. Fractal distribution is not dependent on them. But are defined by the correlation integral [8]. In order to overcome these difficulties we use the mechanism of multifractals. The n-dimensional space is explored and the random quantity ξ is observed. Its many realisations form the distribution that is explores. It can have a self-similar structure and the so-called enthropic dimensions are used to define it - d_q, where $q \geq 0$ is a non-negative parametere. The dimensions d_q are defined the following way: that part of space, containing the explored distribution is divided into pieces with a rim Δ. The probability of falling into i-piece is marked by $p_i = p_i(\Delta)$, $i = 1,\ldots, n$, where $n = n(\Delta)$ is the complete number of pieces with non-zero probability. The function $\Phi_q(\Delta)$ is used:

$$\Phi_q(\Delta) = \left(\sum_{j=1}^{n} p_j^q\right)^{\frac{1}{1-q}}, \quad q \geq 0,\ q \neq 1 \quad \text{and} \quad \Phi_q(\Delta) = \exp\left\{-\sum_{i=1}^{n} p_i \ln p_i\right\}, \quad q = 1. \qquad (4)$$

If when $\Delta \to 0$ there is a limit, $$d_q = \lim_{\Delta \to 0} \frac{\ln \Phi_q(\Delta)}{\ln 1/\Delta} \qquad (5)$$

then we can call it enthropic d_q- dimension in the realisation of ξ. In this case the distribution is self-similar because its characteristic $\Phi_q(\Delta)$ is rising when $\Delta \to 0$ to the power-law. When q=0 $\Phi_0(\Delta) = n(\Delta)$ and d_0 coincides with the fractal dimension, defined by (1). When the probability is one and the same $p_i = 1/n$, $i = 1, 2,\ldots, n$, with random q all the functions $\Phi_q(\Delta)$ are equal; $\Phi_q(\Delta) = \Phi_0(\Delta) = n = n(\Delta)$. As a statistic assessment of the quantity $\Phi_2^{-1}(\Delta)$ in the realisation of N vector $\{\mathbf{x}_1,\ldots, \mathbf{x}_N\}$ the *correlation integral* $C(\Delta)$ is used:

$$C(\Delta) = \frac{2}{N(N-1)} \sum_{i,j} H\left(\Delta - \left|\mathbf{x}_i - \mathbf{x}_j\right|\right), \qquad (6)$$

where $H(z)$ is the function of Heviside, equal to 1 when $z \geq 0$ and equal to 0 when $z < 0$, and the summation is done by the all pairs ($\mathbf{x}_i$, $\mathbf{x}_j$). If the number is scale invariant, then $C(\Delta)$ is presented by the power-law: $C(\Delta) \propto \Delta^{D2}$, where D_2 is a *fractal dimension..*

2. Methods of investigation and Results

In our present work the emphasis is put only on the spatial self-similarity of the seismic conditions. There is also a detailed study of the coordinates of the earthquake epicentres. An synthetic catalogue was created and all points in it are arranged in a straight line.(uniform and by random generation). Gradually numbers of points are created with coordinates x,y, but in all cases the straight lines have one and the same length. A specially designed FORTRAN's program is used. There is an extensive use of a cumulative curve as well. (fig.1) Estimating the possible distances is irrespective of time. Fractal coefficients are derived in two possible ways: defining the straight-line part of the cumulative curve in a double logarhythymic graph, and by seismologic software Zmap [9]. The results are shown in table 1. The missing quantities are marked by a hyphen. The remarkable here is the relatively insignificant change of D_2 quantities, irrespective of the number of points on the line. The difference in these quantities is due to the peculiarities of the software used. For completely generated points the closeness of D_2 to one can be owing to a lack of fractal properties.

Table.1.*The values of fractal dimension of a different number of points on a straight line with the same length. (take a look at the text)*

N	H=0				
	$\mathbf{D_2}$ (Zmap)	**err**	$\mathbf{D_2}$ (for)	$\mathbf{D_2}^{\mathbf{RND}}$(Zmap)	**err**
200	0.93	0.01	0.85	0.96	0.01
250	0.99	0.01	0.89	0.97	-
300	0.95	0.01	0.89	0.97	0.01
400	1.00	0.01	0.88	0.97	-
500	0.97	0.01	0.88	0.97	-
800	0.97	0.01	0.88	0.98	0.01
1000	0.98	0.01	0.88	0.96	0.01

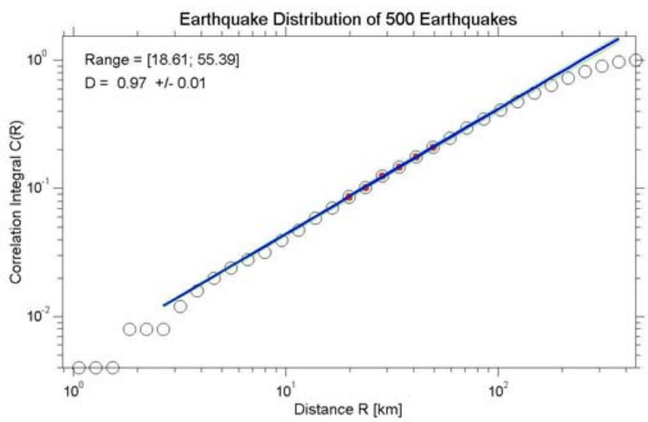

Fig.1 *Example of use correlation integral to estimate fractal dimension.*

In comparison with these data we can point out the fractal dimension of an earthquake catalogue in Morgan Hill, California. Unlike the simulated data, here investigating the hypocentral and epicentral distribution of different magnitude intervals does the estimations. Fig.2 shows epicentral maps of two different magnitude intervals. It is obvious that the linear interval of that seismic area wide 5-6 km and long about 60 km, the depths vary between 0 and 14 km. This influences the values in table 2.

Table.2 *Fractal coefficients for a real earthquake catalogue Morgan Hill* [9].

M_{min}	N	H≠0		H=0		M	N	H≠0		H=0	
		$\mathbf{D_2}$	err	$\mathbf{D_2}$	err			$\mathbf{D_2}$	err	$\mathbf{D_2}$	err
1.5	3422	1.44	0.02	0.97	0.01	[1.0, 1.5)	4446	1.70	0.04	0.98	0.02
2.0	1410	1.40	0.02	0.95	0.01	[1.5, 2.0)	2013	1.43	0.02	0.97	0.01
2.5	578	1.25	0.04	0.89	0.02	[2.0, 2.5)	833	1.40	0.02	0.93	0.02
3.0	206	1.14	0.12	0.78	0.01	[2.5, 3.0)	372	1.13	0.02	0.85	-
3.5	71	0.86	-	0.59	0.01	[3.0, 3.5)	135	0.70	-	0.81	-
4.0	15	(1.15)	0.04	0.70	0.02	[3.5, 4.0)	56	0.78	0.02	0.86	0.01
						[4.0, 6.0)	15	(1.15)	0.04	0.70	0.02

All the earthquakes with a magnitude up to M_{min} = 3.0 form a flat self-similar picture; D_2 changes between 1 - 2. For the epicentral distributions but D_2 is less then one. The values of the fractal coefficients vary respective of the density of events in space.

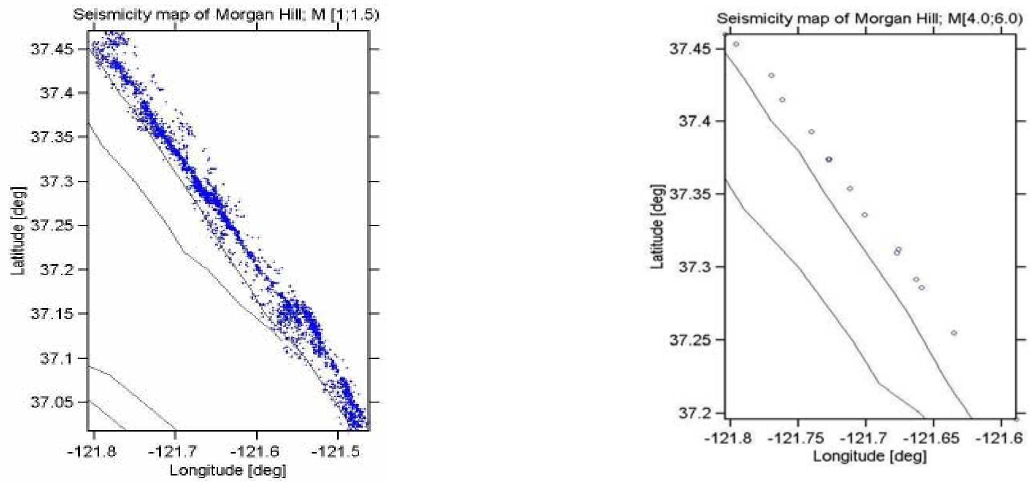

Fig.2 *Epicentral map of Morgan Hill for two magnitude intervals.*

As for events with a magnitude M > 3.5 in both cases the fractal coefficient is < 1, with the exception of the strongest earthquakes where $D_2 = 1.15$. This result cannot be explained with one sentence because there is a relatively small depth interval-from 6.5 to 12km. D_2 measures the degree of fractal grouping of points in space, marking the narrow clusters. If the distribution of points is completely random and unpredictable in 2D-space, D_2=2. Falling to a lesser degree means that the distribution of points will group in self-similar structure. The results of that investigation clearly show that such estimations could continue for a long time and be very profound. To make explicit conclusions about the fractal coefficients of real earthquake catalogues in different magnitude intervals, it is necessary to proceed further investigations of other similar areas.

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